NETWORK MODELS FOR CYBERCAR TRANSPORT

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Name of responsible: Martin Lowson
E-mail: martin@bris.ac.uk, martin@lowson.net

University of Bristol
Advanced Transport Group
Faculty of Engineering
Bristol BS8 1TR
England

Abstract:

A possible model system for public transport based on future cybercars is evaluated. The work considers an idealised system to meet uniform trip demand patterns over a city. The system proposed may have value in its own right, but is introduced to enable the effectiveness of meeting transport demand with transport networks of different density to be calibrated

Models are presented for trips by idealised corridor and network transport. The results for both cases were found to be very similar. Minimum trip times, including walk, wait, in-vehicle and transfer elements, were found to occur with a station separation of around 0.5 km. Maximum average speed was found to be around 15 kph.

The network transport was based on a model grid based synchronous system which could serve a whole city with a maximum of one transfer. This model has been proposed by others, but as far as is known no basic results have been published for its operational effectiveness. The analysis has provided a number of mathematical results for this system.

An interesting result, which may be new, is that the average route length in a grid based city is equal to one sixth of the city perimeter served, independent of grid density.

The results demonstrated that transport effectiveness increased with reducing vehicle size. Optimum vehicle size for 0.5 km spacing was projected to be 8. This would require automatic control to be effective. This makes a case for the consideration of cybercars for public transport

Keyword List:
Fleet management

1. Introduction

The objective of this study is to introduce and evaluate a possible model system for public transport based on future cybercars. The work considers an idealised system for uniform trip demand patterns. The system proposed may have value in its own right, but is introduced as a model system to enable the effectiveness of meeting transport demand with transport networks of different size to be calibrated.

Ideal transport networks are normally determined by prescribing the demand expected and solving an optimisation problem for the routing using linear programming techniques. While this approach can give useful results in specific circumstances, it does not provide generalised results. Indeed, general rules for transport networks appear to be few and far between. This is due to the strong link between the detail of the routing pattern and the detail topography of any real city.

It is believed that there are fundamental laws controlling transport which are concealed by the detail issues of practical realisations. Use of idealised models is expected to be of value in bring out these fundamental issues which could otherwise be concealed by such detail.

The results should be of value in determining the basic effectiveness of a transport system as a function of network density and transport vehicle size.

2. A Linear Model

2.1 Basic Model

The problems of collective –corridor transport are established. Any corridor can only serve trips which are along that corridor. Collective transport requires both waiting and frequent stops, probably at every stop on the route during peak periods. This may be evaluated via a simple model. The model assumed is shown in Figure 1. The corridor transport stops at each of the stops, assumed to serve a square area with side equal to the distance between the stops.

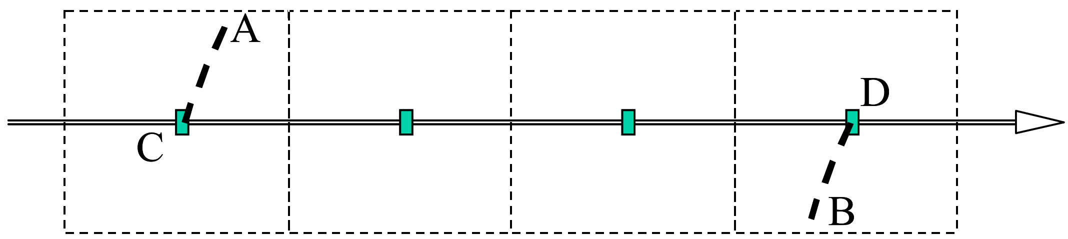


Figure 1 Area Served by Corridor Transport

A trip from start at A to destination at B requires:

1. Walk to station A-C
2. Wait for transport C-C
3. Stop at every stop C-D
4. Walk to destination D-B

The present model involves an estimation of the times taken for each part of the trip. A version of this argument was presented in Lowson (2002)

2.2 Walk and Wait Times

Average walk time to the station is dependent on size of the area served by the station, which is in turn dependent on the average stop separation. A simple assumption is that the corridor is serving a "grid" city with all roads laid out at right angles. Although not typical of all European Cities, this offers an acceptable approximation for the purposes of the present estimates.

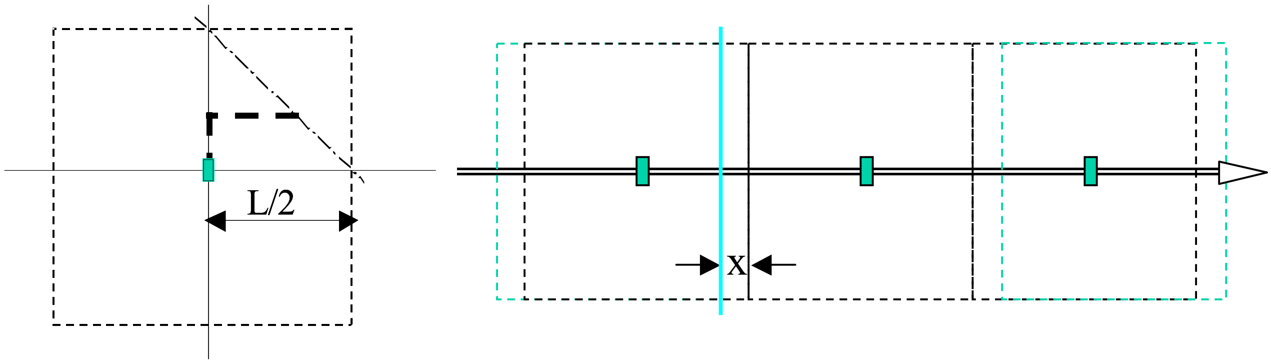


Figure 2 Diagrams showing walk trip length

Figure 2 shows this typical case. A walk from any location to the central station will involve a trip N-S and a trip E-W. Consider a trip starting from any point on the diagonal line. The length of any trip from a point on this line to the centre is L/2 where L is the length of side of the square. But by symmetry since there is exactly the same area on the far side of the line away from the station as on the near side, this line also represents the average trip length.

Thus, the average walk length in a grid route system over a service area of side L is simply L/2. If it is assumed that the walk trip has to be made at both the start and end of the journey then the average distance walked is identically equal to the average stop separation L.

Use of any form of public transport involves a walk at each end of the trip. In typical cases such as shown in Figure 1, the area served by each station can be assumed to be at the centre of gravity of the served area. Thus the average distance from all points in served area at the start to all destination points in the served area at the destination is equal to the station separation. This is an interesting result which applies to a wide range of circumstances; for example, it applies both to grid based and to straight line travel.

Since, under the above fairly general assumptions, for a corridor model the average distance between start and destination is simply the station spacing, the walk required to get to and from the station is a necessary overhead. Although some walks are in the direction of travel, others are in the reverse direction, while half of all walk distance is normal to the direction required. This overhead adds to the average time taken for travel, but not to the distance usefully travelled.

If it assumed that passengers will walk to the downline station where this provides a net benefit in travel time, there is a small modification to the above argument. This is illustrated in the second diagram in Figure 2. Suppose that the blue line indicates the boundary between the locations where it is preferable to walk to the upline or downline stations. Then on the boundary the journey time via either station is the same, either by walk directly to the downline station, or by walk to the upline station and in-vehicle travel to the downline. This can be expressed algebraically as

$$T = (L/2 + x)/W = (L/2 - x)/W + L/V$$

Where W is the walk speed and V in the vehicle speed (which should include the effect of stops).

This gives $x = L/2 \,.\, W/V$

The effect is that the area served by any station is displaced upline. Under the present grid city assumptions it can be seen that the additional walk time to be added on for upline passengers is balanced the reduced walk time to be added for the downline. Thus the average walk distance to the station remains the same. However, the area served has been displaced upline by x. Similar arguments apply to the passengers arriving at the destination, who can choose to get off one stop early. Thus at the destination, the area served is displaced downline by x. This means that the average distance between origin and destination served by a station pair a distance D apart increases to D + 2x, ie to

$$D + LW/V$$

This only makes a small difference to the numerical results, but is included for completeness.

In practice bus or other journeys will use variable spacings so that the relations above will not apply exactly. However, it appears to offers an acceptable first approximation for the walk distance required. Walk times can be found directly from the walk distance by assuming an average walk speed, taken here as 4.8 kph ie 80m/min the average walk speed recommended by the Confederation of Passenger Transport.

In addition to the walk time there is also a wait time. For the present calculations, this has been assumed to be 5 minutes. This would imply a service frequency of 10 minutes, only occasionally provided by conventional transport.

Finally, a typical trip length must be assumed. For the purposes of the present comparisons, this has been taken to be 8 km, corresponding to the average trip length in the UK. As noted above the average separation of origin destination pairs served by stations 8 km apart is equal to 8 + LV/W . The total time is the time taken in-vehicle plus the walk overhead at both ends of the trip, plus the wait time. The average speed is found by dividing total distance by total time as defined.

2.3 Average Speed In-Vehicle

It is of interest to start with the in-vehicle speed for the central part of the trip. The results are shown in Figure 3. They are based on a simple Newton's Law calculation of the acceleration – deceleration process from stop to stop. It is assumed that acceleration and deceleration occur at 0.1g and that stops are 20 seconds each. These results parallel results given originally in Hamilton and Nance (1969) and Lowson (1999).

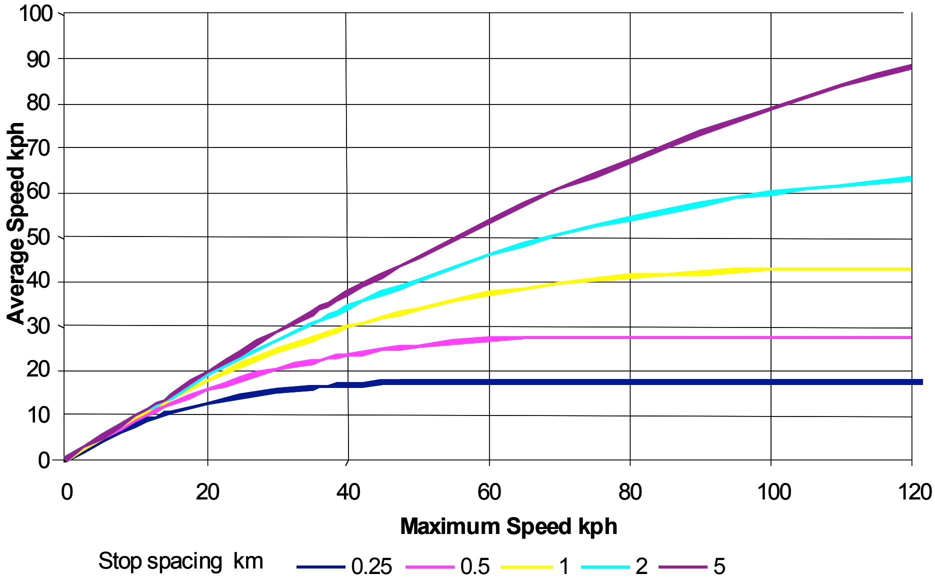


Figure 3 Average Speed in-Vehicle Against Maximum Speed for Various Stop Separations

Stop to start times on buses, including door opening, passenger alighting and door closing can be as little as 10 seconds. However passenger boarding normally takes rather longer, especially if there is a need to pay fares to the driver. For light rail times very low stop times are less likely to be achieved since the driver has less direct interaction with the boarding process. Measurements on buses over several routes in Cardiff showed that the average stop time was 23 seconds between 9.00 and 12.00. Other measurements in peak periods in Bristol showed that average stop times could often be over 30 seconds. Thus it is thought that 20 seconds is an acceptable and conservative overall figure. But in any case, modest changes in stop time have little effect on average speed compared to the deceleration acceleration process.

Figure 2 shows the average speed achieved for various stop spacings. It can be seen that high maximum speeds are of little benefit if stops are closely spaced. Under these circumstances, the vehicle merely accelerates to the mid point between the stops and then decelerates without reaching its maximum speed. For 250m stop spacings, the average speed achieved is less than 20 kph regardless of handbook maximum speed.

This corresponds to speeds achieved in practice by buses in favourable conditions. Light rail, or other systems such as monorails and Automatic People Movers (APMs), which have a higher maximum speed, will normally use longer stop spacings, reducing accessibility in order to provide higher average trip speed. Even so it can be seen that the average in-vehicle speeds achieved for 1 km stop spacings is still only 40 kph.

2.4. Results Including Walk and Wait

Figures 4A and B give the results of these fuller calculations. The two Figures show results for bus and light rail respectively. For the bus case, an average in-vehicle speed of 30 kph has been assumed. This is a reasonable assumption for achieved in-vehicle speed in a city where the bus is obliged to stop regularly at pedestrian crossings, traffic lights etc. The second case shows the results for a higher speed service assumed here to be 80 kph. This is a somewhat generous figure to represent light rail, monorail or APM. This figure also provides an indication of the possible effects of priority bus lanes, or guided bus, which could provide increases in in-vehicle speed for buses.

The results in both Figures 4 are presented in terms of average speed achieved against stop spacing. The top curve gives the speed achieved in-vehicle, and is essentially a replot of the 30 kph results from Figure 2. At high stop spacings, it is possible to achieve high in-vehicle speeds, approaching the maximum speed of the vehicle being considered. However, the addition of walk and wait elements to the journey reduces overall trip speed considerably.

As might be expected that the best overall speed for the journey is achieved when stop spacings are short and the amount of time spent walking to and from the stop in is minimised. It can be seen that for the bus case this provides an optimum stop spacing of around 0.5 km. This is quite close to the average stop spacings used by buses in city operations, although typically closer stop separations (and thus lower average speeds) will occur in the city centre.

For the Light Rail/APM model, the optimum stop spacings are found to be around 0.75 km. The higher speed of the vehicle means that a higher proportion of the time is spent in the walk for the optimum case.

However the most striking feature of these graphs is the low average speed achieved, for the bus this is 14.0 kph and for the Light Rail/APM 17.4 kph. This is because the length of time in the walk part of the trip forces the systems to work at short stop spacings for which the in-vehicle speed is of little benefit. The small improvement in average speed offered by the far higher maximum speed of the Light Rail/APM case is striking. [1] It is also noteworthy that these average speeds are virtually identical to the average speeds achieved by cars in peak periods. This speed is achieved on the corridor, which itself only serves a limited proportion of the trips desired. It is not surprising that current forms of public transport have little attraction compared to car transport.

[1] Doubling maximum speed again to 160 kph (or indeed again to 320 kph) provides no benefit. The maximum achieved overall speed is 17.5 kph.

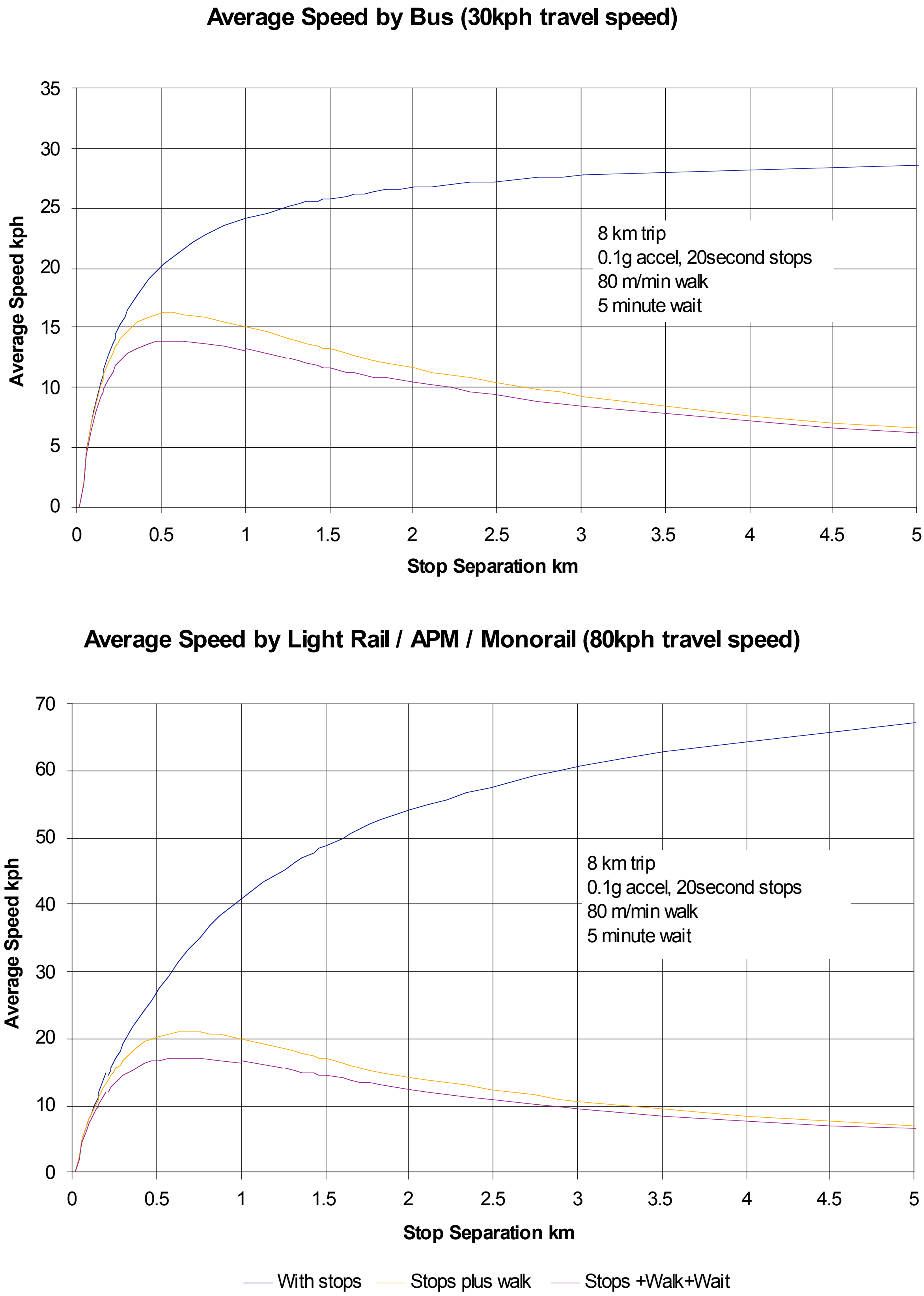


Figure 4 Overall Average Speed for Conventional Transport vs Stop Separation

3. The Network Model

3.1 The Basic Model

For the present purposes it is necessary to develop a model which reflects demand over a complete city, rather than within a single corridor as analysed in the last section. Many cities demonstrate a trip demand density which increases towards the centre of the city. In the past smaller single centre cities had trip patterns which were dominated by the demand for trips to and from the centre. This is far less prominent today. Cities now are essentially mulit-centre, with a dispersed travel demand. Cox (2002) points out that the demand for travel to the centre rarely exceeds 10% of total trip demand in any city. Indeed, it is recognised that urban decay has led in some cases to "doughnut" cities in which the demand for trips concentrates on a ring some distance from the old city centre. Specific modelling of particular types of city may be undertaken as a second phase of the present work. However it is suggested that the assumption of a uniform demand over the complete city is an acceptable first approximation to many existing transport patterns. More importantly for the present purposes, it provides a simple starting point for analysis. It is believed that in practice details of demand patterns will only have second order effects on the results.

The Model City

European cities have a complex cellular structure, and transport results can be very specific to the particular city topology being considered. US cities are generally laid out on a grid pattern, at least in part. The grid pattern provides a helpful starting point for the present work since it simplifies the analysis usefully. It also provides a direct link to the linear model considered earlier.

The model transport system to be studied is for a city of prescribed size, in which all transport links lie on a square grid, running either N-S or E-W. This is shown in Figure 5. There is little additional loss of generality by considering a square city.

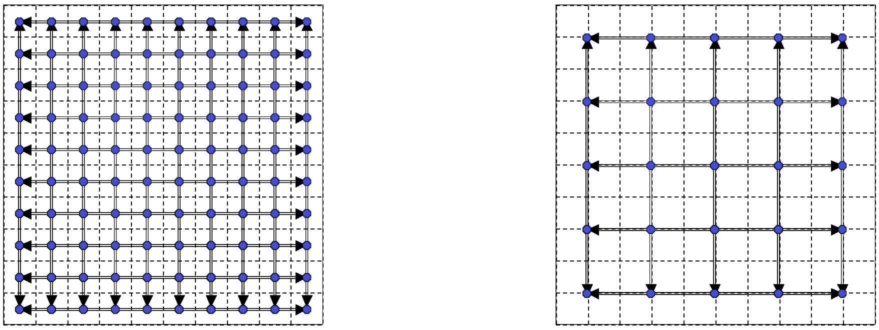
The Model Transport System

An idealised system has been specified to meet the demand. This follows ideas that have been put forward by several others, for example Cox (2002).

The system assumed consists of a series of vehicles which traverse the city bi-directionally, either North South or East West. Stops are located at each intersection at which transfers are permitted. Thus it is possible to get from any point in the city to any other with just one transfer. For the purposes of the present investigation, it is immaterial whether the routes are formed from bus, rail or indeed airborne links

In the example system shown in Figure 5a the square city is divided into 100 blocks 10x10 with a station at the centre of each block. This means that the length of each double track is nine blocks.

At each station, it is presumed to be possible to board a vehicle travelling N, S, E or W. It is also assumed that transfer is possible between NS and EW at each station. Stations at an edge have a restricted set of transfer possibilities.



a) 10x10 b) 5x5

Figure 5 Example Track Coverage

This appears to be a practicable system for many cities. It can be realised by a conventional bus or train, especially if the route network is segregated from other uses. However, a more effective realisation of the system could be fully automatic cybercars. This would permit the use of smaller vehicles since costs would not then be dominated by the cost of the driver.

There are a variety of issues in the design of such a system. Many of these are associated with detailed engineering, for example how to achieve effective transfer from line to line. However, for the present purposes such issues will be ignored. The principal concern of the present study is the determination of optimum scaling parameters. These include

1. Number of lines required to cover city. In figure 1a above a total of 20 tracks with 40 lines are shown. A larger mesh version of this is shown in Figure 1b. This shows a 5x5 matrix system with 20 lines. It is clear that the larger mesh system will involve longer walk times for the passengers, and large vehicles for their carriage. The optimisation of this is less clear.

2. Number of vehicles required on each line. For example in Fig 1a if it were desired to offer a transfer opportunity on arrival at every station then nine vehicles would be required on each line, each stopping at every station at the same time. Use of a smaller number of vehicles will result in less transfer opportunities. The issue is the trade off between lower cost and better passenger service.

3. There is also the question of the size of vehicle required to meet the demand under the various assumptions.

Answering the above questions will provide basic information of the relative benefits of basic scaling parameters on passenger service.

3.2 Characteristics of the Basic Network

The two key decisions for this network model are the choice of network density and the associated choice of numbers of vehicles. This generates a series of interesting results. The issue of numbers of vehicles will be examined first of all.

Consider a generalised network of the type shown in Figure 5. Vehicles run both North/South and East/West on each route.

Suppose that there are n bi-directional routes in each direction.

This means that each intersection will service an area equal to the whole area of the city divided by n^2.

The simplest realisation of the proposed synchronous network is to have n-1 vehicles running in both directions on each line. On the assumption that each takes the same time to get from stop to stop then each vehicle will stop at every stop at the same time thus permitting convenient interchange for the passengers with minimum waiting time.

Under these circumstances, the number of vehicles required is 4n (n-1). This may be a practicable system if the network is relatively sparse.

A variation of this system is to run with half the vehicle numbers on each line. If the reduced number of vehicles were launched in phase at the edges of the city then the effect would be that for half of the stops vehicles would arrive with no other vehicle available for transfer, thus requiring passenger waiting. For this particular layout a small modification solves this problem. If alternate routes are launched out of phase then every vehicle continues to arrive at every intersection at the same time as all other vehicles reach the same intersection. The transfer process occurs at each set of stops alternately. (cf Figure 6A below) Under these arrangements, there is again no delay in transfer for the passenger.

Unfortunately, no equivalent solution occurs for smaller numbers of vehicles. In effect the decision to be made is how much delay to have between successive vehicles. This delay will be assumed an integral multiple of the time taken to transit between stops. The first case described above implies 1 stop time between every vehicle. The second case implies two stop times between every vehicle. For smaller numbers of vehicles with greater distances between vehicles some delay in transfer for some passengers is inevitable.

An interactive spreadsheet program has been written to establish the level of delay occurring for different cases. Diagrams giving the key results are shown in Figures 6.

The base diagram is as shown below. The centre of each five by five block represents the stop, shown by an X. The clock time when vehicles arrive is shown to the North of X for vehicles running North, to the East for vehicles running East and so on. Clock time is based on the phase relative to a zero at the SW corner. All vehicles traverse the distance from one station to the next in the same time, so that time is measured as the integral numbers of single station traverse time. All arrivals in synchronisation with the SW have zero phase.

	Delay North to West		Delay North to East	
Delay West to North		Clock time for North running		Delay East to North
	Clock Time for West Running	**X**	Clock Time for East running	
Delay West to South		Clock Time for South Running		Delay East to South
	Delay South to West		Delay South to East	

If a passenger arrives at a station wishing to transfer he may have to wait for the next vehicle to arrive. The delay will be an integral number of station traverse times. The transfer delays are shown for the eight possible cases eg North to West and North to East etc. The average delay at any station is the sum of the eight delays for each combination divided by eight. The average delay over the whole system is the average of all these delays.

The same format is used for each case. The fully synchronous 1x1 case is trivial and is not shown since it is simply a zero in every box. The 2x2 case was discussed earlier. The solution is straightforward and is shown in Figure 6A. This shows that vehicles leave and arrive at the bottom left or top right stations in all directions with zero phase delay, and at the top left and bottom right with a delay of unity.

This pattern can be repeated as required to fill a space with any number of rows/columns via a tiling process.

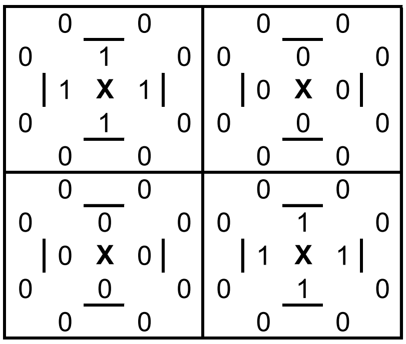


Figure 6A Timing Diagram for 2x2 Matrix

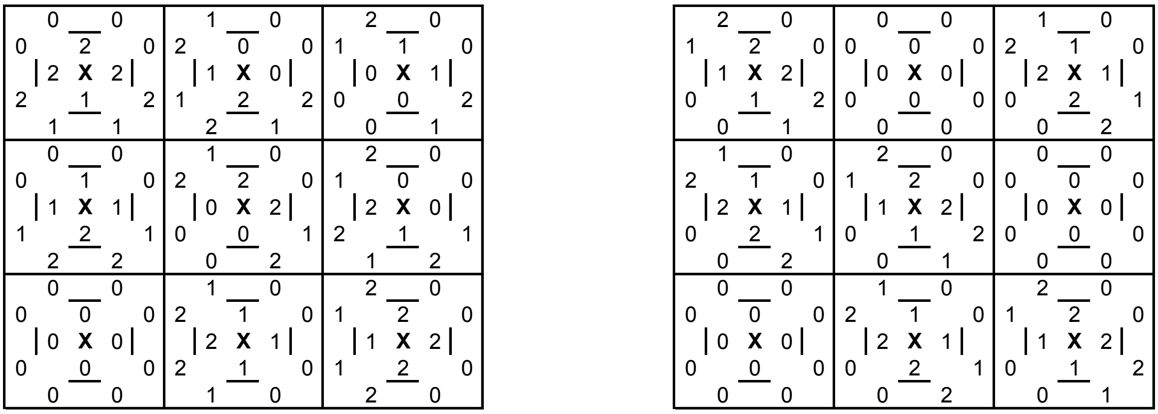


Figure 6B Timing Diagrams for 3x3 Matrix

The 3x3 case does not give a zero transfer time result. Two examples are shown for the three by 3x3 case with different vehicle phasing. The first is the obvious phasing process of delaying each departure successively by one stop. This does not provide the minimum transfer time result over all possible route changes. By phasing the departure of the vehicles in each direction, (second diagram in Fig 6B) it was found possible to provide a minimum solution.

Cases for 4x4 and higher orders have also been studied. It appears that the minimum overall time delay for all possible transfers on the system is obtained when the phase for successive routes on the system increase in one direction and reduce in the other. It has not been proven that this is a minimum solution but by inspection of the lower order cases this does appear to give a satisfactory result. Further even if not absolutely optimum it is clear from inspection that the results do approach a minimum overall transfer time.

So far the results can be applied to any scale of network. In effect the 2x2 or 3x3 is a tile which can be used to fill a total area with vehicles separated by 2 or 3 stations respectively. Any real city will have edges. These involve further analysis.

An example of a network with edges is shown in Figure 7. This is a 3x3 tile on a 4x4 network. The best solution for a 3x3 tile is to operate this is a 4x4 (or 7x7) city so that the edge blocks have identical parameters. Importantly, this also means that the NS and EW going vehicles can be turned round without any need to wait when they reach the edge. In Figure 7 vehicles are launched a step apart at successive tracks going either N-S or E-W. The relevant phase is shown by the number immediately next to the centre box in each direction.

The overall delay occurring in a network with edges will differ from the tiled results developed above. Specifically, any vehicle travelling along an edge route will have no options to turn out of the network. Thus any predicted delays from this turn have to be discarded. Equally there will be no vehicles entering the network looking to make turns, so these predicted delays must also be discarded. The effect is shown in Figure 7 by shading on the invalid delay figures at the edge cells.

For the case shown with identical delay matrices on NS and EW edges the effect is that one complete set of N, S, E and W entries into the network are discarded. This means that the total number of delays in this 4x4 network is simply the delays summed over a reduced 3x3 network. This is a general result and will apply to any size of network in which the two edges have the same phase relations for the vehicles. Eg for 4x4 or 7x7 networks for a 3 vehicle spacing, and for a 5x5 or 9x9 network for a four vehicle spacing.

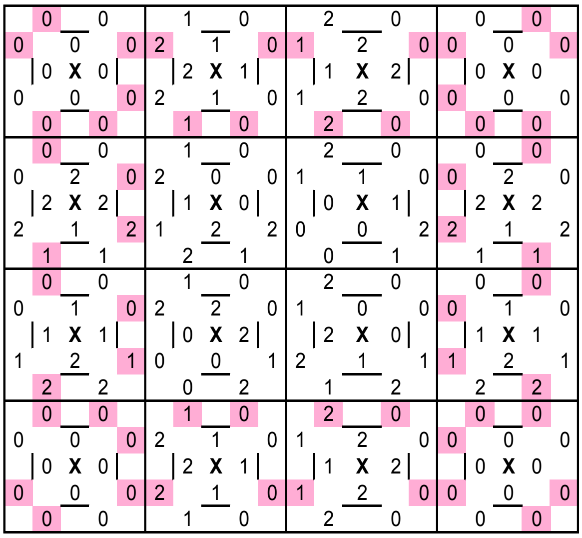


Figure 7 A Network with Edges

This result allows figures for the average delay over a network of any size to be computed straightforwardly. Using the spreadsheet program the total delay over a series of networks of different sizes has been summed. This gives the integral number of delays on all transfers on the network. The average delay can be found by dividing by the total number of transfers. The results are shown in Table 1. Results are given for the total number of delays over the whole network and for the average delay for two cases, the fully tiled case and the case with an edge. The average delay figure gives the average delay which is experienced over the whole network as a multiple of the transfer time between stations.

As discussed above, the case with an edge uses the same number for the total delay but divides by a one order larger area. Thus the result under 3x3 with an edge corresponds to a 4x4 network with vehicles at a spacing of 3.

Network Size	1x1	2x2	3x3	4x4	5x5	6x6	7x7	8x8	9x9	10x10
Total Delay	0	0	36	64	200	288	588	768	1296	1600
Average delay (tiled)	0.00	0.00	0.50	0.50	1.00	1.00	1.50	1.50	2.00	2.00
Average delay with edge	0.00	0.00	0.28	0.32	0.69	0.73	1.15	1.19	1.62	1.65

Table 1 Average Delay due to Transfer as a function of Network Size

The results are slightly surprising. It can be seen that the average delay for the tiled case is always an integral multiple of 0.5. This is presented as a purely numerical finding. Presumably this result could be proven mathematically.

3.3 Estimation of Vehicle Size

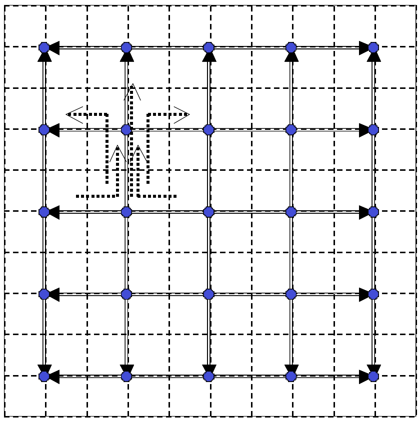


Fig 8 Possible Vehicle Routings

An important issue in the present study is determination of the size of vehicle necessary to meet the various scenarios. This requires an estimation of the likely number of passengers. Both this and average route length can be determined from an examination of the routes assumed.

In the current models it is assumed that there is an equal demand from all stations to all other stations. Using this model it is possible to estimate the typical number of passengers on a vehicle at any point. Figure 8 shows possible passenger routings for northbound passengers in a particular northbound route segment. It is assumed that passengers will take the most direct route to their destination. With one transfer only this can be either a NS route followed by an EW route or vice versa. For the present models there is no difference between these two routings so either could be chosen freely. It can be seen that there are five route sets which would involve passing over the particular segment considered.

The most obvious route set is passengers who leave from a station directly south of the segment and wish to travel to a station directly north. In this case there is no need for transfer and all these passengers would use the segment. In this case there are three stations south of the segment with the option of travelling to two stations north of the segment. Thus the total demand on the segment shown is 6 times the average station to station demand over the whole network.

Passengers from any of the other stations south of the segment, both East and West of the transfer station could chose to traverse this segment to travel to a station to the North of the segment. In this case there are a total of 12 other stations South of segment and two stations directly to the north. The total demand for this segment from this class is potentially 24 times the average demand from on any one station to any other station. However all these passengers have the alternative of going EW then North (as counted above) or North and then

EW to their final destination, in which case they would not traverse the relevant section. The overall demand must be halved to take these two alternative routings into account.

The other set of passengers are those who do choose to travel North and then EW. These will all come from the 3 stations which are due south of the segment; they can choose to travel to any of the 8 stations which lie to the north of the segment, ie not including those directly to the north which have already been included in the first class considered.

The relevant number for the present calculation is the projected peak passenger load. It will be seen that overall the combination of the last two classes of passenger will always total $n(n-1)/2$. In addition the number of direct NS passengers will peak at $(n^2/4-1)$ for an even number of stations and $(n+1)^2/4$ for odd numbers of stations.

The peak number of passengers is therefore

$$P = d\{n(n-1)/2 + (n^2/4-1)\} \qquad \text{n even}$$
$$P = d\{n(n-1)/2 + (n+1)^2/4\} \qquad \text{n odd}$$

Where d is the average demand from station to station over the whole network per unit time. The relevant unit of time is the time for successive vehicles to arrive at the station. A sparser network will have a longer collection time, while vehicles travelling at higher average speeds will have a lower collection time

The demand from a single station to all other stations is dn^2. Overall demand over the whole area requires multiplying by n^2 again. Thus the overall demand for the entire network is $D = dn^4$. For a fixed demand D over a prescribed area (eg a city) the average passenger requirement per vehicle is

$$P = D\{n(n-1)/2 + (n^2/4-1)\}/n^4 \qquad \text{N even}$$
$$P = D\{n(n-1)/2 + (n+1)^2/4\}/n^4 \qquad \text{N odd}$$

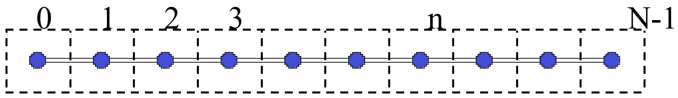
This assumes that every station is being served regularly. If the number of vehicles is reduced by using a higher vehicle spacing than the vehicle size would have to be increased in proportion.

The actual size of vehicle to serve this demand would have to be somewhat larger in order to provide for the random nature of the demand. If it is assumed that the arrival process is of Poisson type then a standard result is that the mean square variance of the process is equal to the mean. Thus the root mean square variance σ is equal to the square root of the mean. If it is desired that there is better than 3σ chance (ie 90%) of meeting the demand then it is necessary to provide additional capacity on each vehicle, giving a total capacity required of

$$P_{3\sigma} = P + 3P^{0.5}$$

3.4 Average Route Length

Calculation of the average route length involves further algebra. It is convenient to first of all evaluate the average route length for a simple straight line route as shown below, this corresponds to East trips only



Number of Routes N-1 N-2 1 0

Consider N stops which will be numbered from 0 to N-1. The number of East going routes from 0 is N-1, and from 1 is N-2 etc as shown. The total number of East going routes available is therefore the sum of the routes from all possible leaving locations. Summing this series gives

Total number of East going routes = N (N-1) /2

Assume a unit spacing of the stations. Then the length of these routes can also be found by summation. The sum of all the routes from 0 is

$$\Sigma_1^{N-1} n = (N-1)N/2$$

The sum of the length of all routes from the m th stop eastwards is

$$\Sigma_1^{N-m-1} n = (N-m-1)(N-m)/2$$

The sum of all routes from all stops is the sum over all values of m of the expression given above ie

$$\Sigma_1^{N-1} (N-m-1)(N-m)/2$$

This expression may be evaluated using standard summation formulae for series to give

Total length of East going routes = (N-1) N (N+1) /6

This result can be generalised for the trips on a rectangular grid. Suppose the total number of stops North South is M. For each row routes will involve all the East trips within that row plus the other M-1 trips which involve either going North or South and then East (or vice versa). In each case the total Easterly component of the trip is the same. Thus the total number length of Easterly routes from departing from a location at any point in one row is simply M times the result given above. The total length over all rows requires multiplication by M again. Thus the total length of East going trips in an MxN network is

$$M^2 (N-1) N (N+1) /6$$

There is the same number of Westerly going routes, so that the total routes E and W is simply double the above.

There is a total of MN start points in an MxN matrix. Each of these provides service to (MN-1) other points. Thus the total number of trips is MN (MN-1)

Dividing gives the average length of all E and W routes as

$$M (N-1) (N+1) / 3 (MN-1)$$

For North South routes the same result applies with M and N reversed ie

$$N (M-1) (M+1) / 3 (MN-1)$$

A single route will involve a combination of NS and EW trips. Thus the total length of route can be found by addition

ie $$M (N^2 - 1) + N (M^2 - 1) / 3 (MN-1)$$

This simplifies to the surprisingly simple result for an M x N network with uniform load

Average route length over all routes = (M+N) /3

It may be noted that the area <u>served</u> has sides of length M and N so that the non-dimensional result is that the average route length is 1/6 of the perimeter of the area served. This result is independent of the stop spacing

The number of neighbouring stops may be shown to be 4MN-2N-2M

Note that the semi perimeter of the network itself is (M+N-2) so that if it is desired to work in terms of lengths of the actual network the nice result of independence of route length on stop spacing no longer applies.

For a square network the result shows that the average route length is two thirds of the side of the area served. This result will be used in determining average trip time.

There are a number of limitations on the general result. It applies to trips taken via the network only. For sparse networks the shorter trips solely within a sub area will be automatically excluded. Also the result provides an equal weighting between long trips and short trips. In practice there will be a distribution of trip size around a median length. Further the shortest trips will almost certainly be undertaken by walk rather than by any form of public transport. Nevertheless the general result provides a useful simplification

3.5 Calculation of Results

Trip Times

There are three components of trip time, walk(both ends) wait and the trip itself

i) Walk Time

The assumption for the present models is that that the operation is in a grid based city, with n tracks N-S and E-W and each of n^2 stations serving n^{-2} of the whole area. Under this assumption each sub area is c/n where c is the overall dimension of the (square) city. If the station is at the centre of this area the average walk distance is c/2n. The walk time is c/2nW where W is the speed of walking. This applies to both ends of the trip to that the average distance walked is simply c/n and time c/nW.

It is interesting to recognise that the average walk distance for this generalised type of transport is exactly equal to the average stop spacing. This is the same result as in the corridor case.

ii) Wait time

The wait time is governed by the spacing of vehicles on each route. The station spacing is c/n, so that if the vehicle spacing is s (ie s=3 implies that vehicles are three station spacings apart). Thus the wait time is sc/nV where V is the average speed of the vehicles between stops (see further discussion below)

iii) Trip time

The analysis presented above showed that, surprisingly, the average route length over all trips is independent of the grid density. Thus under the present assumptions the average (in vehicle) trip length will be the same in all cases. The trip time in vehicle is simply equal to this average distance divided by the average vehicle speed V ie

$$2c/ 3V$$

In addition the time taken for any transfer must also be included. This can be assumed to be a uniform figure for walking between the vehicles within the station plus waiting time. Average waiting time for transfers were presented in Table 1, and are a function of network design / vehicle frequency. The base stop to stop vehicle time is a function of average vehicle speed. The average time from stop to stop vehicles includes time taken for acceleration, deceleration and in station time, and so the speed calculation is identical with the calculation presented in section 2.2. and Figure 2. If vehicles are spaced s apart then the average time between vehicles will increase by a factor s. The Transfer time T_T is given by

$$T_T = F\ sc/nV$$

Where F is the transfer time multiple taken from Table 1. In the present calculations the "tiled" value has been used. This will be edge effects which will slightly reduce the average transfer time experienced over the whole network, but these are not significant for the present purposes.

Average Vehicle Speed

The average speed of the vehicles is a function of the average separation between stations. As already considered for the corridor case, vehicles have to speed up and slow down for each station, and stopping time in the station also directly affects overall trip times. This was shown in Figure 1. It could be agued that overall waiting time in stations will be higher for a network because of the transfer passengers. However for present purposes the corridor results will be taken over for the present network calculations.
A choice also has to be made of vehicle maximum speed. For the longer trips increasing maximum speed does result in a reduction of overall trip time. However for shorter distances the average speed is dominated by the acceleration-deceleration process. A maximum speed of 80 kph has been taken for the present calculations. In practice this only provides any benefit for the longer station separations.

The overall result for trip time is thus

	Walk	Wait	In Vehicle	Transfer
T =	c/nW +	sc/nV +	2c/3V +	Fsc/nV

Simplifying

T = c [{1/nW+s(1+F)/nV + 2/3V)]

Where

c	overall (square) city size in km
n	number of tracks N-S and E-W
W	walking speed (80 m/min)
s	Vehicle spacing
V	Average vehicle speed (taking account of station spacing)
F	transfer time Factor (Table 1)

It will be noted in the formula above that for this model walking waiting and transfer times are all reduced on a small grid. Only the in vehicle time will show a benefit from a larger grid (ie stop) spacing.

4. Results

Initial results have been calculated for the case

City size: c = 10 km This is a modest size city. In Europe typical population densities are around 4000 / sq km. Thus this corresponds to an area with a population of around 400,000. Note that this means the average length of all trips is the city is 3.33 km.

Number of tracks n: this has been taken as 2, 5, 10, 20, 40. This corresponds to station spacings of 5, 2, 1, 0.5, 0.25 km respectively

Vehicle spacing s: particular values of this have been selected, corresponding to the station spacings. This is shown in the Table 2 below. The first two columns give the base number of tracks and associated station (or track) separation. Running a service vehicles offering trnassfer at every stop would require the base vehicle numbers shown in column 3. To provide a complete service with a vehicle stopping at every station simultaneously in all directions is possible for the sparser networks but would require a ridiculously large number of vehicles for the smaller networks. The value of s has therefore been taken to leave the number of vehicles at a reasonable level on the denser networks. The fifth column shows the consequent number of vehicles actually used for each case.

Number of tracks n	Station separation km	Base vehicle numbers	Vehicle spacing s	Number of vehicles
2	5	8	1	8
5	2	80	2	40
10	1	360	5	72
20	0.5	1520	8	190
40	0.25	6240	10	624

Table 2 Input data used in calculations

The results of the calculations are shown in Figure 9 and Table 3. They show that at large station spacings (ie small number of tracks) trip time is dominated by walk time. The large spacing tracks provide a poor service. At smaller separations ie denser track network trip times reduce, with a minimum in this case occurring at around 0.5 km. This figure is very parallel to the results from the corridor routes and it is of interest to compare these results. This is done in Figure 10.

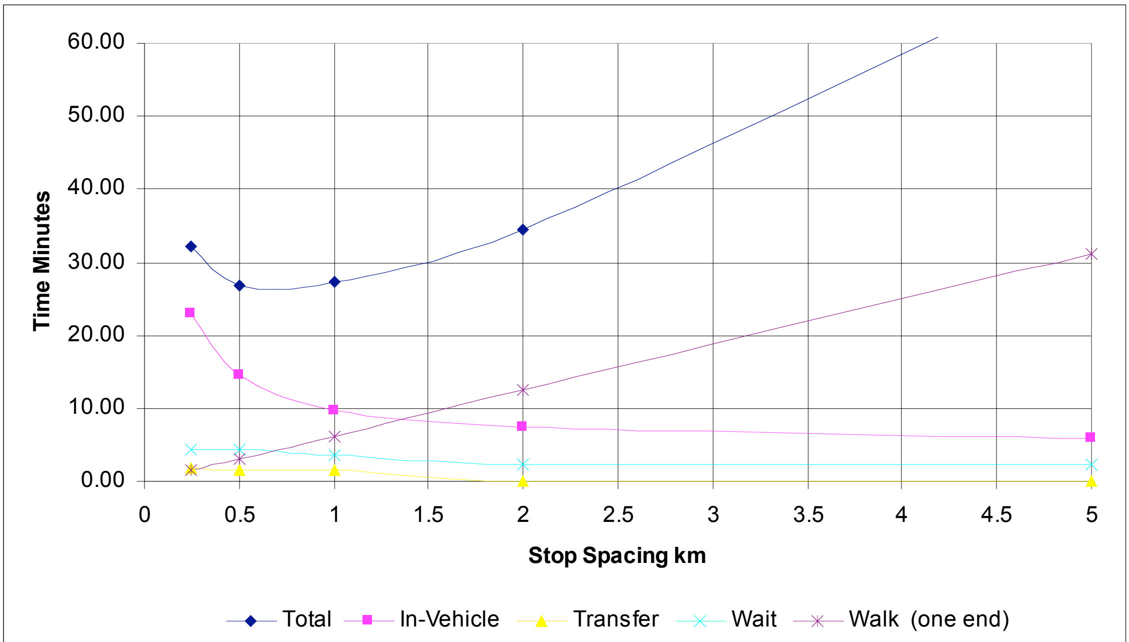


Figure 9 Trip Time for Network Transport for Average 6.6 km Trip

Stop Spacing km	Total	In-Vehicle	Transfer	Wait	Walk (one end)
5	70.68	5.95	0.00	2.23	31.25
2	34.58	7.37	0.00	2.21	12.50
1	27.35	9.74	1.46	3.65	6.25
0.5	26.70	14.48	1.63	4.34	3.13
0.25	32.26	23.08	1.73	4.33	1.56

Figure 3 Output Data for trip time components (minutes)

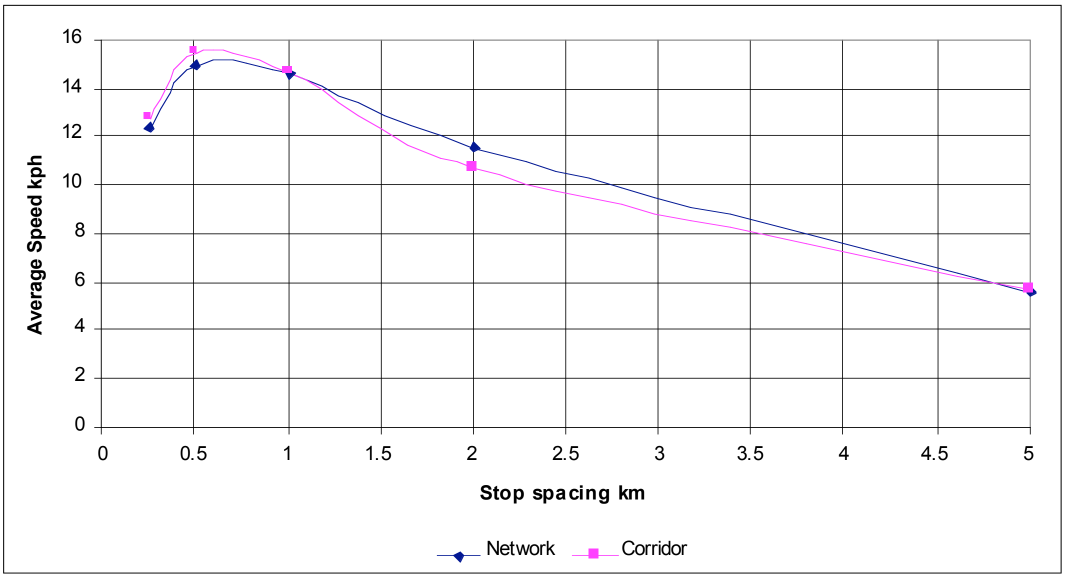


Figure 10 Comparison of Average A-B Speeds on a Network and Corridor System

This figure compares the average A-B speed achieved in the two systems, including all components of trip time viz walk, wait, in-vehicle and (where appropriate) transfer. Both corridor and network figures are based on the same average 6.67 km trip length. It will be seen that the results are virtually identical. The differences are due to a slight difference in assumed waiting time, taken as 5 minutes for the corridor case but at the value shown in Table 3 for the Network case, and the need to add in transfer times for the denser network results.

Several points emerge

1) The broad results of the corridor and network models are very similar.
2) In both cases the best delivered A-B speeds are low, around 15 kph.

3) This is because, in both cases, the trip times at higher separations which offer better in-vehicle speeds are dominated by walk time
4) The network model has additional trip overhead for transfers at the lower spacings

Vehicle parameters projected for the various cases are shown in Table 4 below. This is based on an assumed demand level of 600 trips per square km per hour during the peak hour. This is the actual figure for Bristol and is representative of European Cities. It is assumed that the network is designed to carry 20% of that load.

Track Separation km	Number of Tracks	Number of vehicles	Required Vehicle Size	Seats offered
5	2	8	78	625
2	5	40	42	1698
1	10	72	20	1412
0.5	20	190	8	1606
0.25	40	624	3	2169

Table 4 Vehicle Parameters for the Network Cases

The first and second columns give the separation and number of N-S and E-W tracks. The third column shows the number of vehicles required to serve the network. This takes account of the vehicle separations chosen for this case (previously noted in Table 2). The required vehicle size in Column 4, is estimated from this demand per unit time, the stop to stop time in each case and includes the 3σ statistical correction to cover peak loads. Multiplication of column 3 by column 4 gives the number of seats offered shown in column 5.

The results are of interest in several ways. In essence the two track network provides an approach comparable with existing light rail systems both in max speed and size. It also offers apparently good efficiency in terms of seats provided to do the task. However it can be seen from the previous results that the trip times offered by this system are far too low to provide a practical or attractive transport solution. The denser networks require lower vehicle sizes, with the optimum network under this calculation requiring a vehicle size of about 8.

Generally it will be seen that transport systems with smaller vehicles on denser networks offer greater transport effectiveness. This is believed to be a general result. It will also be noted that the smallest vehicles will require t some form of automatic control to be cost effective. This provides a justification for the cybercar approach.

Cox (2002) analysed the costs of a similar system based on 0.5 mile track separation serving a 20 mile square city. He concluded the annual capital and operating costs of a bus based system would be around 20% of the personal income of the metropolitan area and for rail based 110%. This estimate takes no account of the possible major reductions in cost of both vehicles and infrastructure that can result from use of a small scale system.

The densest networks (0.25 km) do not provide efficient use of the vehicles. Most of seats are being made available to deal with peak loads. Average passenger loads are less than unity. It will be seen that the automatic transfer process proposed is very ineffective for the densest networks. Since typical passenger loads are less than 1 there seems little point in going through the multiple stop process since few passengers will transfer. There appears to be a strong case for other approaches such as PRT to meet demand on dense networks. These would offer zero wait and transfer time and much faster in vehicle speeds because there is no need for intermediate stops.

5. Conclusions

Models have been presented for trips by idealised corridor and network transport. The results for both cases were found to be very similar. Minimum trip times, including walk, wait, in-vehicle and transfer elements, were found to occur with a station separation of around 0.5 km. Maximum average speed was found to be around 15 kph.

The network transport was based on a model grid based synchronous system which could serve a whole city with a maximum of one transfer. This model has been proposed by others, but as far as is known no basic results have been published for its operational effectiveness. The analysis has provided a number of mathematical results for this system.

An interesting result, which may be new, is that the average route length in a grid based city is equal to one sixth of the city perimeter served, independent of grid density.

The results demonstrated that transport effectiveness increased with reducing vehicle size. Optimum vehicle size for 0,5 km spacing was projected to be 8. This would require automatic control to be effective. This makes a case for the consideration of cybercars for public transport

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