B2, An Alternative Two Wheeled Vehicle for an Automated Urban Transportation System

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Abstract-- **This paper describes the preliminary control work on a two-wheeled vehicle called B2 whose wheels belong to the same geometric axes. Closed-loop control is necessary because the passengers stand above the wheels, so that B2 behaves like an inverted pendulum. Using a simple model, two different control laws are compared. The first uses standard linear control techniques to achieve dynamic stability. Its most notable characteristic is that it stabilizes the vehicle inclination about the dynamic equilibrium by minimizing motor effort. A second control law approach, named PDC (Parallel Distributed Compensation) [1], is then investigated. This control law uses a fuzzy model and is another way of stabilizing the cabin angle and controlling the vehicle's speed. Simulations presented show the effectiveness of each control method.**

I. Introduction

We study a very fundamental question related to automated vehicles. Specifically, are there alternative approaches to the basic automobile platform which are more suitable for the 21st century? With recent advances in digital computers, nonlinear control theory, and actuation technology, it becomes possible to consider radically new concepts. For example, a two wheeled vehicle may be safer for the occupants while simultaneously being more agile to negotiate narrow city streets. Furthermore, the reduced volume and lower mass of this configuration would increase fuel efficiency and overall functionality. However, because such a vehicle would be inherently unstable it would require an automatic control mechanism to provide dynamic balancing. In this way, such an automobile could form the basis of an advanced automated transportation system [2] [3].

The nature of this two-wheeled vehicle poses several interesting control questions. For instance, while a person occupies the vehicle, their mass changes the center of gravity for the vehicle, as shown in Figure 1. Here, $\delta = \delta(t) \neq 0$. Consequently, if the standard inverted pendulum control were to be applied, the control algorithm would be obliged to maintain a steady acceleration to achieve the control objective ($\psi \to 0$) [4] [5] [6]. Instead, a control algorithm must be found which seeks the natural equilibrium point of the system $\psi = -\delta$ although δ is unknown. Another problem is that there are restrictions on the method of control. For instance, energy pumping although successful in several applications [7] [8] would be undesirable and impossible because the inclination angle of B2 is restricted to ($\psi \leq 35°$). Finally, besides these considerations, robustness to uncertainty plays a substantial role in the success of B2. The weight and character of the occupants will never be known. In part, they represent unmodeled dynamics; when it stops suddenly they move forward. On the other hand, they are true disturbances since they are autonomous agents able to perform an action independent of B2's dynamics. Given these observations, our paper considers two different control strategies.

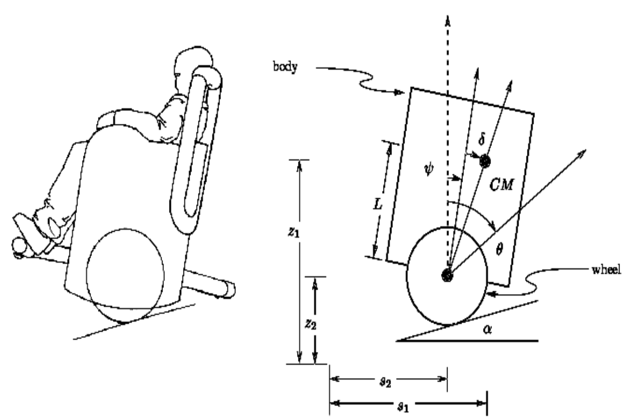

Figure 1: B2 carrying a passenger along an incline of slope α and a simple model of the vehicle. Here, the occupants mass causes an imbalance in the vehicle center of mass (CM) .

First, in a traditional fashion, we develop a simple linear controller with feedback feedforward structure comprising an autobalancing algorithm for the vehicle inclination

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stabilization and a physics based feedforward control for velocity tracking. This control law is unique to the literature in two ways. One, the inclination algorithm integrates the control effort instead of angular error to achieve virtually zero DC motor effort. Implicitly, this algorithm solves for the maximum potential energy of the pendulum, locating the natural equilibrium angle regardless of the passengers posture. Two, a physics based speed algorithm capitalizes on the pendulum instability caused by gravity. Essentially, vehicle is tilted in the direction of desired motion. In response, the vehicle accelerates in that direction suitable to prevent itself from falling over. Once sufficient data has been collected, and the plant characterized, it will be possible to apply alternative controllers which make full use of the identified plant. For instance, a robust control method like μ-synthesis may be applicable [9].

Secondly, in anticipation of the need for greater robustness, we develop a state space fuzzy law [1] [10]. We begin by deriving a non-linear fuzzy Takagi-Sugeno (TS) model [10]. Since the plant equations are affine with input, we achieve perfect matching in a compact set of the premises. To overcome the large number of rules common to fuzzy control, which are problematic to implement, we prove that two rules are sufficient to obtain necessary robustness.

II. Dynamic Model of the Vehicle

A. Nonlinear Equations

Figure 1depicts B2 in a typical situation. Here, a person of unknown mass is being conveyed along an incline α. As can be seen, the equilibrium angle is not perfectly upright, but instead tilted by a small angle δ dependent on the individual. Neither is known to the control law. Therefore, we propose a simple nonlinear model which encapsulates only these particular details. For the present, we assume that α=0 and ignore practicalities like backlash in the gear system, sensor noise, etc. The model of the B2 is derived from Lagranges equations [11] and is described by two second order differential equations of wheel angle, θ, and body angle, ψ:

$$\ddot{\theta}(t) = \frac{1}{\det A}\Big(\big(t_1\cos(\psi(t)+\delta)+t_2\big)\big(t_3u - t_4\sin(\psi(t)+\delta)\big) + t_5\big(t_1\sin(\psi(t)+\delta)\dot{\psi}(t)^2 + t_4\sin(\psi(t)+\delta)\big)\Big) \quad (1)$$

$$\ddot{\psi}(t) = \frac{1}{\det A}\Big(-\big(t_1\cos(\psi(t)+\delta)+psi2\big)\big(t_3u - t_4\sin(\psi(t)+\delta)\big) + \big(psi5 - t_1\cos(\psi(t)+\delta)\big)\big(t_1\sin(\psi(t)+\delta)\dot{\psi}(t)^2 + t_4\sin(\psi(t)+\delta)\big)\Big) \quad (2)$$

with

$$t_1 = M_bLr\sec(\delta) \qquad psi1 = t_1 = M_bLr\sec(\delta)$$
$$t_2 = J_b + \sec(\delta)^2 M_bL^2 \qquad psi2 = J_w + M_Wr^2 + M_br^2$$
$$t_3 = \eta\tau_t \qquad psi3 = t_3 = \eta\tau_t$$
$$t_4 = M_bgL\sec(\delta) \qquad psi4 = t_4 = M_bgL\sec(\delta)$$
$$t_5 = \sec(\delta)^2 M_bL^2 + J_b + J_m\eta^2 \qquad psi5 = J_m\eta^2$$

and, in both equations,

$$\det A = d_2\cos(\psi(t)+\delta) - d_3\cos^2(\psi(t)+\delta) + d_1$$

with

$$d_1 = J_m\eta^2J_b + J_m\eta^2\sec(\delta)^2M_bL^2 + J_w\sec(\delta)^2M_bL^2 + J_wJ_b + J_wJ_m\eta^2 + M_wr^2\sec(\delta)^2M_bL^2 + M_wr^2J_b + M_wr^2J_m\eta^2 + M_b^2r^2\sec(\delta)^2L^2 + M_br^2J_b + M_br^2J_m\eta^2$$
$$d_2 = 2J_m\eta^2M_bLr\sec(\delta)$$
$$d_3 = M_b^2L^2r^2\sec(\delta)^2$$

Where M and J are the linear mass and moment of inertia respectively. Where the subscripts b, w, and m indicate the term is associated with either the body, wheel, or, motor. The terms L, r, η and τ_m are the height of the CM, wheel radius, the gearbox ratio, and, motor torque constant respectively. The plant input is, u, the motor current.

B. Linear Model

Equations (1)(2) are linearized by assuming small angles ($\sin(\delta+\psi)\approx\delta+\psi$, $\cos(\delta+\psi)\approx 1$, and $\sec(\delta)\approx 1$) and $\dot{\psi}\approx 0$. Consequently, two transfer functions are derived. One from the motor input to the cab inclination, ψ(t):

$$\psi(s) = \frac{a_\psi\delta(s) - b_\psi u(s)}{s^2 - a_\psi} \quad (3)$$

The other from motor input to wheel angle θ(t):

$$\theta(s) = -\frac{1}{s^2}\left(\frac{a_\theta a_\psi}{s^2 - a_\psi} + a_\theta\right)\delta(s) + \frac{1}{s^2}\left(\frac{a_\theta b_\psi}{s^2 - b_\psi} + b_\theta\right)u(s) \quad (4)$$

with the coefficient $a_\psi > 0, a_\theta > 0$ and $b_\psi > 0, b_\theta > 0$.

C. TS Fuzzy Model

The TS fuzzy model allows for an exact representation of nonlinear models with affine control input, on a compact of the state variables [13]. The main benefit for using this model is that it provides a systematic framework to design control laws [14]. Unfortunately, for this approach the stability criteria are only sufficient, such that numerous model rules are necessary to meet conservative specifications. Furthermore, the number of rules grow by 2^n where n is the number of non-linearities [13]. Still, it has been used successfully in real time control of an inverted pendulum, for example [15], and also in simulation for a double inverted pendulum [16]. Thus, it is reasonable to consider this approach for the control of B2 dynamics.

The continuous TS fuzzy model is written as [10]:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{r} h_i(z(t)) C_i x(t) \end{cases} \quad (5)$$

Where $x(t) = [\theta(t) \quad \psi(t) \quad \dot{\theta}(t) \quad \dot{\psi}(t)]^T$, $z(t)$, $y(t)$ and $u(t)$ are the state, premise, output, and input vectors. Here the premise vector is independent of the input and often considered as a part of the state vector or as a linear combination of this one. $F_j^i : (j \in \{1, 2, \cdots, p\})$ are the fuzzy sets. The weighting functions $h_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))}$, $w_i(z(t)) \geq 0$. $w_i(z(t)) \geq 0$ represent the significance associated to the i[th] rule. For this paper, the fuzzy model is supposed to be locally controllable ($\forall i \in \{1,\ldots,r\}$ the pairs (A_i, B_i) are controllable) and locally observable ($\forall i \in \{1,\ldots,r\}$ the pairs (A_i, C_i) are observable). In fact, this is guaranteed by proper design of B2.

To derive the TS model, we choose $\delta = 0$. The state vector is given by: $x(t) = [\theta(t) \quad \psi(t) \quad \dot{\theta}(t) \quad \dot{\psi}(t)]^T$ where the positions are measurable.

As commented earlier, the number of model rules goes as 2^n with n nonlinear terms [13] [14]:

$\sin(\psi(t))$, $\cos(\psi(t))$, $\cos^2(\psi(t))$ and $\sin(\psi(t))\dot{\Psi}(t)^2$

Thus, n=4 indicating 16 rules are required. However, with some compromise the number of rules can be reduced to 2 while maintaining model [13] [16].

First, by first order Taylor Expansion we can rewrite two of the nonlinear terms as:

$$\frac{\sin(\psi(t))}{\psi(t)} = \frac{\Psi_0 \sin(\psi(t)) - \psi(t)\sin(\psi_0)}{\psi(t)(\Psi_0 - \sin(\psi_0))}.1 + \frac{\Psi_0(\psi(t) - \sin(\psi(t)))}{\psi(t)(\Psi_0 - \sin(\psi_0))}.\frac{\sin(\psi_0)}{\psi_0}$$

and

$$\cos(\psi(t)) = \frac{\cos(\psi(t)) - \cos(\psi_0)}{1 - \cos(\psi_0)}.1 + \frac{1 - \cos(\psi(t))}{1 - \cos(\psi_0)}.\cos(\Psi_0)$$

Whose membership functions are bounded in the range $\psi(t) \in [-\Psi_0, \Psi_0]$ for $\Psi_0 \in [0 \quad \pi/2]$ implying:

$$\left| \frac{\Psi_0 \sin(\psi(t)) - \psi(t)\sin(\psi_0)}{\psi(t)(\Psi_0 - \sin(\psi_0))} - \frac{\cos(\psi(t)) - \cos(\psi_0)}{1 - \cos(\psi_0)} \right| < 2,4\%$$

Therefore the transformation on $\cos(\psi(t))$ can be eliminated with little compromise and the fuzzy model order reduced to 2^3 or 8 rules.

Secondly, $\cos(\psi(t))$ and $\cos^2(\psi(t))$ can be simplified as:

$$\cos^2(\Psi(t)) = \frac{\cos^2(\Psi(t)) - \cos^2(\Psi_0)}{1 - \cos^2(\Psi_0)}.1 + \frac{1 - \cos^2(\Psi(t))}{1 - \cos^2(\Psi_0)}.\cos^2(\Psi_0)$$

which is also bounded for $\psi(t) \in [-\Psi_0 \quad \Psi_0]$, and $|\psi_0| < 30°$ by:

$$\left| \frac{\cos^2(\Psi(t)) - \cos^2(\Psi_0)}{1 - \cos^2(\Psi_0)} - \frac{\cos(\psi(t)) - \cos(\psi_0)}{1 - \cos(\psi_0)} \right| < 1,8\%$$

(B2 has limited angular range by design). Again, the term $\cos^2(\psi(t))$ can be disgarded with little lose of information. It follows that the fuzzy model is then described by 2^2 or 4 rules.

Thirdly, the term $\sin(\psi(t))\dot{\Psi}(t)^2$ can be disgarded because the large moment of inertia associated with the passengers keeps $\dot{\psi}$ small. Then, the final fuzzy model is described by only two rules. In fact, this approximation has a secondary benefit because we intend to use a fuzzy observer which will have common premise functions with the fuzzy model [14]. As pointed out in [17], the separation principle for observer and controller design is not valid unless the nonlinearities of the premise functions are independent of unmeasured variables. Thus, discarding $\sin(\psi(t))\dot{\Psi}(t)^2$ not only decreases system order, but also aides control and observer synthesis.

Finally, the complete fuzzy model is comprised of two rules:

if $\psi(t)$ is F_1^i then $\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases}$ for i=1,2 (6)

with $F_1^1(\Psi(t)) = \frac{\Psi_0 \sin(\psi(t)) - \psi(t)\sin(\psi_0)}{\psi(t)(\Psi_0 - \sin(\psi_0))}$

$F_1^2(\Psi(t)) = 1 - F_1^1(\Psi(t))$

and premise variable $z(t) = \psi(t)$. The two submodels are described respectively by the matrices A_1, B_1, C_1 and A_2, B_2, C_2 with :

$$A_1 = \begin{bmatrix} 0_{2\times2} & I_{2\times2} \\ \begin{bmatrix} 0 & \frac{t_4(t_5 - t_2 - t_1)}{\det A_1} \\ 0 & \frac{t_4(psi_5 + psi_2)}{\det A_1} \end{bmatrix} & 0_{2\times2} \end{bmatrix} \quad B_1 = \begin{bmatrix} 0_{1\times2} \\ \begin{bmatrix} \frac{t_3(t_1 + t_2)}{\det A_1} \\ \frac{-(t_1 t_3 + psi_2 t_3)}{\det A_1} \end{bmatrix} \end{bmatrix}$$

$C_1 = [I_{2\times2} \quad 0_{2\times2}]$ $\det A_1 = d_1 + d_2 - d_3$

$$A_2 = \begin{bmatrix} 0_{2\times2} & I_{2\times2} \\ \begin{bmatrix} 0 & \dfrac{t_4 \dfrac{\sin\Psi_0}{\Psi_0}(t_5 - t_2 - t_1\cos\Psi_0)}{\det A_2} \\ 0 & \dfrac{t_4 \dfrac{\sin\Psi_0}{\Psi_0}(psi_5 + psi_2)}{\det A_2} \end{bmatrix} & 0_{2\times2} \end{bmatrix} \quad B_2 = \begin{bmatrix} 0_{2\times2} \\ \begin{bmatrix} \dfrac{t_3(t_1\cos\Psi_0 + t_2)}{\det A_2} \\ \dfrac{-(t_1 t_3\cos\Psi_0 + psi_2 t3)}{\det A_2} \end{bmatrix} \end{bmatrix}$$

$C_2 = C_1$ and $\det A_2 = d_1 + d_2\cos\Psi_0 - d_3\cos^2\Psi_0$

III. CONTROL OF B2 INCLINATION AND VELOCITY

A. Linear Control

In this section, the preliminary linear control algorithms of B2 are proposed and justified. This includes an autobalance routine to place the vehicle center of mass above the wheel axle and a feedforward algorithm to control its longitudinal velocity. In planning a control strategy, we acknowledge several facts about B2:

1. The few fuzzy model rules needed suggest that the open-loop plant is quite linear. In fact, this is a typical property of pendulum for moderate values of ψ.
2. All states are measurable (ψ,θ), or easily estimated $(\dot{\psi},\dot{\theta})$ thanks to B2's large inertia. Alternatively, using the digital filter (z-1)/(Tz) to estimate $\dot{\psi}$, has been shown to yield good results [8].
3. The principle unknowns are the cabin mass, M_b, and imbalance angle δ.

Presented with these facts, the most obvious strategy will be to use state feedback theory. The advantages of this technique are its simplicity of design, natural robustness, and easy implementation. Supposing that the closed loop system have the equation:

$$|sI - (A - BK)| = (s + (a_1 + jb_1))(s - (a_1 - jb_1))(s + a_2) \qquad (7)$$

By Ackermanns formula [5] [18] the gain matrix K of A-BK must be

$$K = -\frac{\eta\tau_m}{M_b}\left\{\left[rg + \frac{L}{3}(b_1^2 + a_1(a_1 + 2a_2)\right],\right.$$
$$\left[\frac{Lr}{9g}(4L(a_1^2 + b_1^2)a_2 + 3g(2a_1 + a_2))\right], \qquad (8)$$
$$\left.\left[\frac{Lr^2}{3g}(a_1^2 + b_1^2)a_2\right]\right\} = [K_{p\psi}, K_{d\psi}, K_{d\theta}]$$

where we have imposed several simplications: L>>r, J_b>>J_w, J_b>>J_m, M_b>>M_w, and $J_b \approx M_b L^2/3$.

To minimize coupling between control of cabin inclination and speed a_2 should be choosen such that a_1>>a_2. Or simply that the closed loop cabin angle dynamics are much faster than the closed loop speed dynamics.

1) Autobalance algorithm

During ordinary operation, B2 will have an asymmetric mass distribution as shown in Figure 1. Consequently, if the control objective is $\psi \to 0$, then the vehicle must continuously accelerate at a rate $\ddot{s}_1 = g\tan(\delta)$ to compensate for gravity instability. Of course, the control objective is not to bring $\psi \to 0$. Instead, we wish to automatically balance B2. As an example, if $\delta = \pi/32$, then the natural equilibrium would be $\psi > -\pi/32$; the angle where the center of mass is coincident with the vertical axis through the wheel axle. The eigenvalues of the open-loop plant do not depend on the value of δ. Therefore, the autobalance mechanism should not change the dynamic response of the control law (s away from the origin). On the other hand, as seen in Eqs. (1-2), the value of δ manifests as a DC (s=0) torque on the cabin of B2 when $\psi \neq -\delta$. Consequently, the motor torque (or equivalently the control input u) must exactly compensate the imbalance torque. This suggests a method of autobalancing the B2 wherein the control effort is integrated and subtracted from the control input. This is described as in Figure 2.

The feedforward component a_ψ/b_ψ translates the requested angle with respect to zero to a new angle with respect to -δ (the natural zero angle of the plant). Recall that the dynamic equation for B2 inclination was given in Eq. (3) as

$$s^2\psi = a_\psi\psi + a_\psi\delta - b_\psi u\ .$$

The closed loop system is described by the plant Figure 2. By the Routh-Hurwitz criteria test we find the following are necessary and sufficient conditions for the stability of the closed loop system:

$$a_\psi K_{i\psi} > 0 \qquad b_\psi K_{d\psi} > K_{i\psi}$$
$$K_{p\psi} K_{d\psi} b_\psi^2 > a_\psi K_{i\psi} \qquad b_\psi Kp > a_\psi$$

Finally, if we suppose that CM is located at δ(s)= δ then it is possible to use the final value theorem to accertain the equilibrium angle of B2 as $\lim_{p\to 0} s\psi(s) \to \psi_r - \delta_0$.

Therefore, the control algorithm provides the desired result in that it requires a slight modification to the existing control structure, yet achieves auto balance without prior knowledge of the mass distribution. Additionaly, at the equilibrium state of the system, $\ddot{\psi} = 0$ and $\psi = -\delta$ implies that u=0 from Eq.(3). Thus, autobalancing has the additional benefit that it causes the nominal motor torque to be virtually zero for balancing.

2) Velocity control by feedforward

In the last section, an autobalance routine was devised that caused the inclination of B2 to move towards the natural equilibrium. Finally, it is necessary to control the velocity of B2.

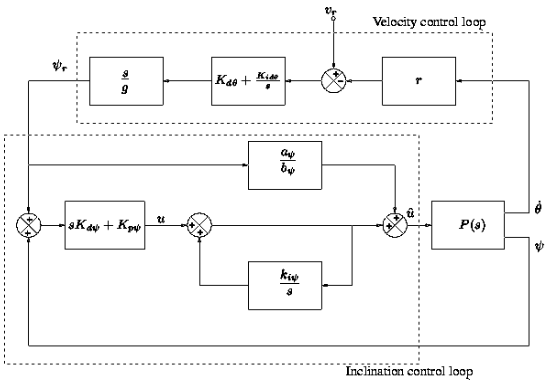

Figure 2: The complete B2 inclination and speed controller for B2. The inclination control loop stabilizes the cabin angle. The velocity control loop governs the speed of B2.

Ha [12] solved for a feedforward velocity command by inverting the state matrix **A**. Essential to his technique was that **A** be invertible, which is only true if lossy terms are included in the model. In the case of a small model or robot, the rolling resistance and internal friction are relatively significant. On the other hand, rolling resistance and motor internal resistance are not very significant in comparison to the inertial forces for B2. Thus, in practice it would seem undesireable to take this approach in the case where there are perhaps several orders of magnitude between the largest and smallest terms of **A**. For this reason, the approach used for control of the robot "Yamabico Kurara" [12] is not applicable to B2. Thus, another method must be used to track velocity. One simple and intuitive technique would be to use the gravity instability of the cabin. In principle, this is accomplished by tilting the cabin by an amount ϕ_r in the direction of travel. To prevent the cabin from falling, the pendulum control must accelerate B2 to precisely compensate for destabilizing force. Once the desired speed has been achieved, the cabin angle ϕ_r goes to $-\delta$, causing the forward acceleration to be zero. The appropriate value of ψ_r can be calculated from a simple static force analysis. It also can be derived from the Eqs. (1)(2) by allowing $\dot{\psi} \to 0$ and making some reasonable approximations (such as $M_b >> M_w$) we find

$$\frac{1}{g}\ddot{s} = \psi + \delta \to \psi_r$$

Where we have made use of the fact that the autobalancing routine cases $\psi \to \psi_r - \delta$ as $t \to \infty$.

As expected, forcing the B2 cabin to tilt causes B2 to accelerate in the direction of tilt. This obvious result forms the basis of the B2 velocity control. Furthermore, the robustness of this approach is obvious by observing that the mapping from ψ_r to $\dot{s}$ does not depend on any plant parameters. Figure 2 shows a complete block diagram of the proposed B2 control system. As indicated, there are two controllers. The innermost system corrects the cabin angle, this is labeled as the *Inclination control loop*. The outermost system is labeled *Velocity control loop* and is responsible for maintaining the speed of B2. As can be seen, actual velocity, $v = r\theta$ of the plant is compared to the desired velocity v_r. Via a PI controller or lag compensator, the resulting signal is used to calculate a reference angle ψ_r for the inclination loop. Justification of the compensator structure is trivial and can easily be seen to stabilize the plant by a root locus method. The choice of lag compensator greatly improves the steady state error of the plant. Of course a lead compensator could be used to enhance the transient response of the speed loop.

B. PDC fuzzy controller – quadratic stabilization, and fuzzy observer

1) PDC fuzzy controller

This approach is based on the second method of Lyapunov, and gives sufficient stability conditions. These conditions are conservative, as they don't take into account the premises part, i.e. only the conclusion part of the rules is used.

The present work uses the classical results of quadratic stabilization. Other works are also available allowing the use of relaxed stabilization conditions [20], or non quadratic Lyapunov functions [21] [22] [23], and also other control laws [23] allowing to outperform the results of the quadratic condition of stabilization.

For a PDC control law, each control rule R^i is obtained according to the fuzzy model. So, the fuzzy controller shares the same fuzzy sets as the fuzzy model and the same weights $w_i(z(t))$. For continuous models, the PDC fuzzy controller is written as [1]:

Fuzzy controller rule $i = 1, 2, \cdots, r$:

If $z_1(t)$ is F_1^i and $\cdots$ and $z_p(t)$ is F_p^i then $u(t) = -F_i x(t)$

or : $$u(t) = -\sum_{i=1}^{r} h_i(z(t)) F_i x(t) \qquad (9)$$

The synthesis of the controller consists of finding the feedback gains of the conclusion parts F_i. Notice that in the case of a PDC approach, the closed loop model is written as:

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) (A_i - B_i F_j)\, x(t) \qquad (10)$$

with the sub-models $A_i - B_i F_j$ such that $h_i(z(t)) h_j(z(t)) \neq 0$ are present. As, in most cases, the premise part is not taken into account in the stability conditions, it will lead in more conservative results due to the coupled sub-models $A_i - B_i F_j$ such that $i \neq j$. With $G_{ij} = A_i - B_i F_j$, the main result is given by the following theorem [1]:

Theorem 1 : The equilibrium of the fuzzy model in closed loop (10) is asymptotically stable if there exists a matrix $P > 0$ such that: $G_{ii}^T P + PG_{ii} < 0$ (11)

$$\left(\frac{G_{ij}+G_{ji}}{2}\right)^T P + P\left(\frac{G_{ij}+G_{ji}}{2}\right) < 0 \quad \textit{for } i < j \qquad (12)$$

for each $i, j \in \{1,\ldots,r\}$ excepted the pairs such that $\forall t$ $h_i(z(t))h_j(z(t)) \neq 0$.

by a change of variables we obtain a LMI formulation to search simultaneously for the matrices $P > 0$ and F_i [13]. The expression of the PDC control law for the B2 is written as: $u(t) = -\sum_{i=1}^{2} h_i(z(t))F_i x(t)$ (13)

The fuzzy controller is found by resolving a LMI problem [13]. In our case, an important parameter is Ψ_0, the maximal value of $\Psi(t)$.

2) Fuzzy observer

Several states of the system are not available for direct measurement necessitating a fuzzy observer to reconstruct these states. With $\hat{x}(t)$ the estimate state vector, $y(t)$ and $\hat{y}(t)$ respectively the final output of the fuzzy model and the final output of the observer, a fuzzy observer is described in the following way [13] [16]:

Rule i, $i = 1, 2, \ldots, r$:if $z_1(t)$ is F_i^1 and…and $z_n(t)$ is F_i^n

then
$$\begin{cases} \dot{\hat{x}}(t) = A_i\hat{x}(t) + B_i u(t) + K_i(y(t) - \hat{y}(t)) \\ \hat{y}_i(t) = C_i\hat{x}(t), \quad i = 1, 2, \cdots, r \end{cases} \qquad (14)$$

A fuzzy observer shares the same fuzzy sets as the fuzzy model and keeps the same weights $w_i(z(t))$. The final output of the observer is given by :

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(\hat{z}(t))\{A_i\hat{x}(t) + B_i u(t) + K_i(y(t) - \hat{y}(t))\} \\ \hat{y}(t) = \sum_{i=1}^{r} h_i(\hat{z}(t))C_i\hat{x}(t) \end{cases} \qquad (15)$$

with $h_i(\hat{z}(t)) = \dfrac{w_i(\hat{z}(t))}{\sum_{i=1}^{r} w_i(\hat{z}(t))}$ and : $\sum_{i=1}^{r} h_i(\hat{z}(t)) = 1$, for all t.

where K_i represent the gain matrices of the fuzzy observer.

In the general case, the complete closed loop model, i.e. fuzzy model, PDC law and fuzzy observer must be analyzed to derive the entire stability of the controlled scheme. It can remain in very difficult conditions to be verified [13].

Nevertheless, an interesting particular case occurs when only measurable variables are used for the premises [16], i.e. when we can replace $h_i(\hat{z}(t))$ by $h_i(z(t))$. Actually, in this case for PDC control laws a separation principle is available [16]. This one can be summarized as follows:

i) Find $P > 0$ and gains F_i for the PDC control law allowing the stabilization of the model without observer, using for example conditions (11) and (12) of theorem 1;

ii) Find $P_{obs} > 0$ and gains K_i allowing the asymptotic convergence of state error for the fuzzy observer. One can use for example conditions (11) and (12) of theorem 1 on the transpose problem, i.e. replace the pairs (A_i, B_i) by (A_i^T, C_i^T).

Clearly, the construction of the fuzzy model of the B2 vehicle has been done to use measurable premises variable $\psi(t)$, so the separation principle is available.

The problem of using a LMI formulation is the difficulty to introduce performance constraints easily, due to the change of variables.

For the observer, we want to guarantee a fast convergence on the non-measured variables, i.e. the speeds $\dot{\theta}$ and $\dot{\psi}$. A quadratic synthesis using classical Riccati equations is then proposed for each sub_model, in order to adjust the convergence dynamic of the state errors. The LMI conditions are then only used to check the convergence of the state error for the fuzzy model, i.e. only to find if a matrix $P_{obs} > 0$ exists. The fuzzy observer gains are then fixed after several trials paying careful attention to the dynamic of the non-measured variables.

IV. Simulations

We now present the simulation results of B2 for the two controllers discussed previously. Unfortunately, at this time, direct comparison between the linear and fuzzy controllers is not possible. So, we present the simulations separately. We begin with the linear control.

A. Linear control of B2

In section III, a simple control law that provides both autobalancing and speed tracking was proposed. In particular, we wish to better understand the effect of important unknown parameters such as δ and M_b. The simulations use a nonlinear model which includes secondary nonlinearities like input and state saturations, and actuator dynamics for instance.

Figure 3 shows the response of B2 to different initial starting angles. Overall, we see that the controller is working as intended: Cabin angle converges to $-\delta$, velocity converges to v_r. In regards to the effect of varying ψ_0, we observe that B2 moves by about 1m at startup. However, the peak longitudinal displacement of B2 increases with increasing initial angle even though the final displacement appears to be invariant to initial angle. Besides peak displacement, cabin overshoot at startup increases as the initial angle increases, though this should not be surprising.

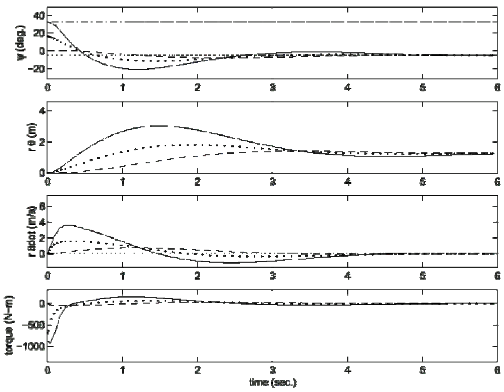

Figure 3: A simulation depicting the effect of different starting inclinations on the startup trajectory of B2. Here, the dashed, dotted, and solid curves correspond to ψ_0=0,16°, 32° respectively. The line at 32 degrees on the topmost plot of ψ(t) represents the larges admitted angle of the cabin. The line at –5° represents the natural equilibrium of the cabin, namely, ψe=-δ. Whereas the dotted line in the velocity plot (third from top), represents the reference velocity, v_r.

Figure 4 and Figure 5 show two families of simulations all with a common control law, wherein δ and M_b are constant unknowns of different values. In the first case, where δ is a constant, the cabin inclination comes to rest at the natural equilibrium of ψ_e= -δ.

Similarly, B2's velocity converges to the reference velocity v_r; the controller is working as designed. Apparently, the final displacement of B2 strongly depends on the imbalance angle, for δ=10°, the $r\theta_f \approx 2m$ while for δ=0 $r\theta_f \approx 0$. Consequently, the eigenvalues of B2 are only slightly sensitive to changes in δ. On the other hand, we expect that the eigenvalues of B2 are more strongly dependent on the mass of the cabin.

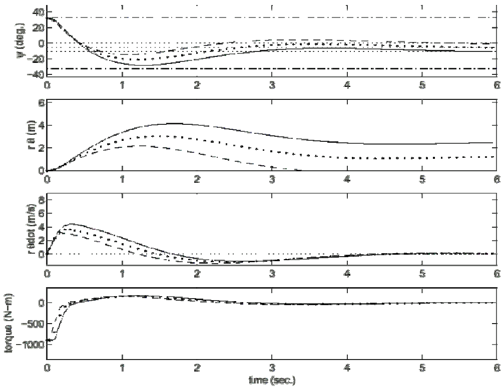

Figure 4: Three simulations illustrating the effect of different imbalance angles at startup. In this plot dashed, dotted, and solid curves correspond to δ=0°, δ_0=5.2° and $\delta=\delta_{max}$=10.4° respectively.

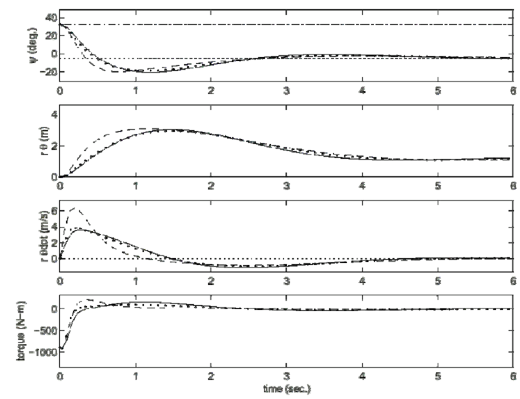

Figure 5: Three simulations showing the effects of occupancy weight. In these simulations, the moment of inertia, and center-of-mass all vary according to the number of individuals occupying B2. In this plot, the dashed curve corresponds to zero occupants, the dotted curve corresponds to 1 person, and the solid curve to 2 persons. The line at 5.2° on the top graph represents the equilibrium value ψ_e= -δ where $\delta=\delta_{max}/2$. The line at 32° on the same plot represents the maximum inclination angle. The fine dotted line on the graph of $r\dot{\theta}$ represents the reference velocity v_r.

Given a few modest assumptions, it is not difficult to show that the open loop poles migrate from $s = \pm\sqrt{6g/h_t}$ with no occupants to $s = \pm\sqrt{3g/h_t}$ with full occupancy. This is born out in Figure 5, wherein the same control shows similar response even as the cabin mass changes from $M_b = M_t$ to $M_b = M_t + 2M_p$, a difference of 180kg. In comparing the effect of changing M_b and δ, we note that even though the eigenvalue of B2 are more dependent on M_b, the closed loop performance of B2 more strongly depends on δ.

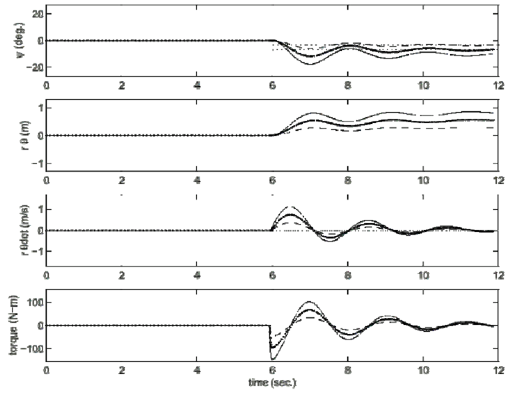

Figure 6: A simulation showing the effect of δ=δ (t) where δ (t) is a step function. Here is varied by an amount δ_{max}(k/3) with k=1,2,3.

Figure 6 shows the results of three simulations of B2 when δ=δ (t). Where δ (t) is a step function of amplitude δ_{max}(k/3) for k=1,2,3. Here we observe that the closed loop system has very poor dynamic response for δ type disturbances. As a result, B2's longitudinal motion increases by larger amounts. The cause is simply that the autobalancing routing is simply an integrator pole at the complex origin (DC frequencies). As the imbalance angles frequency increases, it moves further from the origin, and thus becomes less effected by the integrator pole. Thus, the high frequency components of the response are absent, and the step response poor. On the face of it, there are two approaches to improve response. The first would be to use a larger motor, the second would be to use a lead compensator. Clearly, the former approach is both more elegant and less expensive. Another choice may be to use an adaptive control to estimate δ quickly then cancel its effect with feedforward [18]. Irregardless of the method employed, it is apparant that dynamic autobalancing must be improved to achieve better performance from B2.

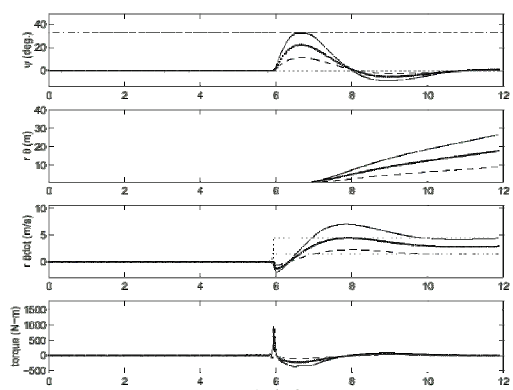

Figure 7: A simulation showing the speed step response of B2 for three different speeds: 1.5, 3.0, and 4.5m/s (respectively dashed, dotted, and solid curves).

The controller must also be able to track a specified speed. Figure 7 shows the speed step-response of B2 for different values of desired speed: 1.5, 3.0, and 4.5 m/s. As can be seen, B2's speed reaches a steady state condition in about 4 seconds for each test case. Note that in the last instance, the vehicle acceleration exceeds 4m/s^2 such that the skids touch down (ψ(t) contacts ground in top plot). B2's response to stopping suddenly was also investigated.

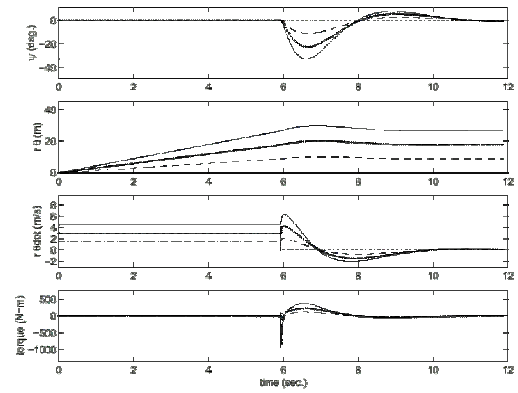

Figure 8: A simulation showing the B2's response to a sudden stoping command with three initial speeds: 1.5, 3.0, 4.5m/s, (respectively dashed, dotted, solid curves).

Figure 8 shows three simulations each with different initial speeds of 0, 1.5, and 4.5m/s respectively. Observe that the autobalancing routine naturally causes B2 to lean backwards into the direction of deacceleration. Besides being necessary, it is a desireable characteristic of the control mechanism since it prevents B2 from flipping over with sudden deaccelerations. Obviously, any landing skids on the B2 must not hinder its ability to tilt. In these examples, brakes are not used to slow B2, instead, the stopping action is completely motor driven. Both the set of simulations for Figure 7 and Figure 8 indicate the presence of a zero in the RHP. This can be seen by looking at the velocity plot, $r\dot{\theta}$, and observing that the actual velocity first decreases then increases in response to a step in requested velocity as is typical with non-minimum phase systems. This will be addressed more later.

B. Fuzzy Simulations:

We now present the simulation results for the fuzzy control scheme. For these simulations, the model rules are chosen for $\Psi_0 = 0,56$ without any constraint as:

$F_1 = [-0,0096 \quad -0,6082 \quad -0,0220 \quad -0,1926]$,

$F_2 = [-0,0153 \quad -0,8778 \quad -0,0352 \quad -0,3073]$.

Figure 9 gives a result for the PDC control law with $\delta = 0$ and the vector of initial conditions of the nonlinear model:

$$x(0) = [0 \quad 0,1 \quad 0 \quad 0]^T .$$

As concern the observer, the results for the B2 vehicle are:

With $Qo = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 1000 \end{bmatrix}$ $Ro = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$

The weighting matrices of the minimized criterion : $\int_0^{\infty} \left(x^T Q_o x + u^T R_o u \right) dt$. For each rule we obtain :

$$K1 = \begin{bmatrix} 1.4177 & -0.0029 \\ -0.0029 & 1.4193 \\ 100.0033 & -0.8252 \\ -0.0009 & 100.2299 \end{bmatrix} \quad K2 = \begin{bmatrix} 1.4177 & -0.0014 \\ -0.0014 & 1.4187 \\ 100.0008 & -0.4157 \\ -0.0002 & 100.1369 \end{bmatrix}$$

Using these matrices we obtain a matrix $P_{obs} > 0$ and then to prove that the complete closed loop is stable using the PDC defined before and the separation principle.

An example of result is shown **Figure 10**, with $\delta = 0$, $x(0) = [0 \quad 0.1 \quad 0 \quad 1]^T$ and $\hat{x}(0) = [0 \quad 0.1 \quad 0 \quad 0]^T$. As $x_2(t)$ is a measurable variable, only $\hat{x}_4(0)$ is supposed to be unknown.

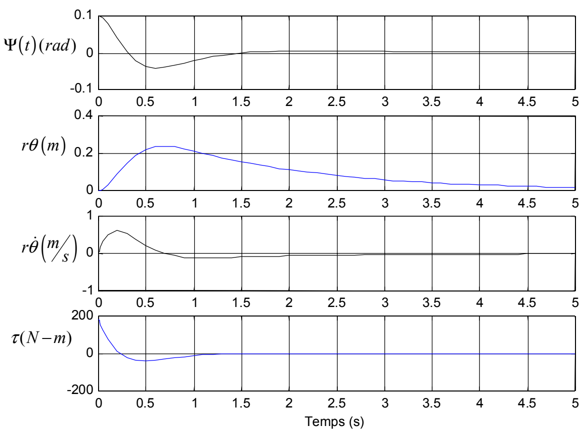

Figure 9 : a PDC control law applied to the B2

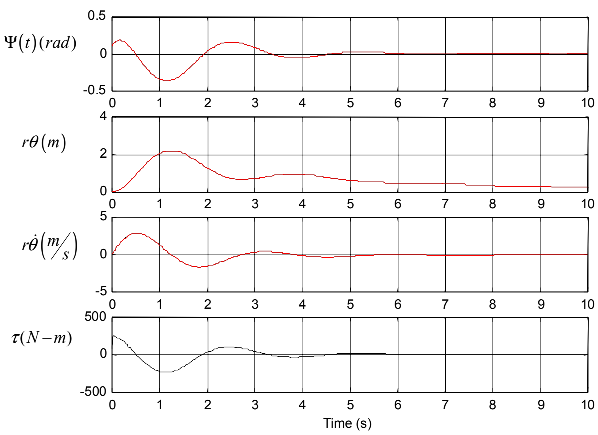

Figure 10: results for a PDC law with a fuzzy observer

Nevertheless, as long as a passenger in the vehicle moves, the center of gravity of the vehicle is no longer verifying $\delta = 0$, as shown in Figure 1, therefore, $\delta = \delta(t) \neq 0$. Then, the control law must try to reach the natural equilibrium point of the system ($\psi = -\delta$).

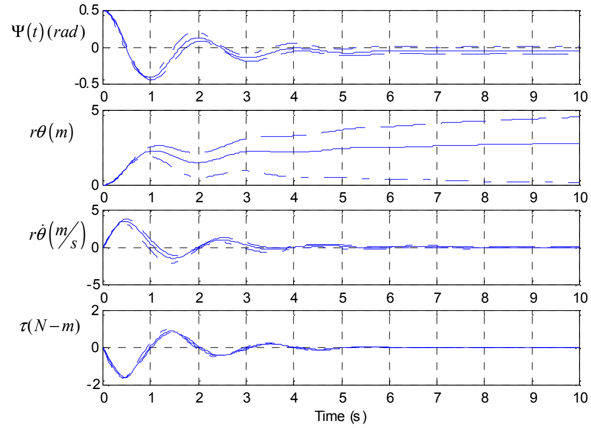

Figure 11: Simulations illustrate the effect of different imbalance angles at startup. In this plot dashdot, solid and dashed curves correspond to $\delta_0 = 0$, $\delta_0 = 0.05rad$ and $\delta_0 = 0.09rad$

V. Summary and Conclusion

In this paper, we have proposed two different control laws for the B2. As seen in the first method, B2 requires two levels of control. First, it must be able to balance itself, $\psi \to 0$. Second, it must be able to track a reference velocity to be a useful vehicle $\dot{s}_2 \to v_r$. Making this task more difficult, the passengers mass and mass distribution are unknown. To surmount this difficulty, a control strategy was presented wherein B2 balances itself without knowledge of the center-of-mass angle δ or M_b. This was accomplished by forcing DC cabin tilt control effort to zero. As was seen, it works quite well for all admissible masses and center-of-mass conditions. However, it was not perfect, when the center-of-mass shifted quickly, B2's position was slightly unpredictable. To achieve velocity control, a feedforward structure was devised wherein B2 tilts in the desired direction. To prevent the cabin from falling over, B2 accelerates forward, essentially using the cabin inertia to compensate for the instability of gravity. Both of these approaches are new to the literature.

The second method presented another type of control law, a non-linear feedback law based on a Takagi Sugeno fuzzy model. Since several states are not available, a fuzzy observer was derived. The first results demonstrating the effectiveness of the approach. Regardless of the control technique employed, the principle thesis of this paper was proven. Essentially, that a two wheeled vehicle can be made to work well under the numerous conditions a vehicle might encounter.

Future work will focus on improvements to the control laws by using constraints on the inputs/outputs. As well as showing robustness and/or performance aspects of the proposed laws. Finally, B2 must be implemented and tested in real world situations.

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